(p,d) Nuclear Structure Studies on Nuclei between $A=12$ and $A=48$*

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The 40-MeV proton beam from the University of Minnesota Linear Accelerator has been used to study a number of medium-mass nuclei via the single-neutron pickup reaction. Elements studied were C^{12}, O^{16}, $Mg^{24,25,26}$, $S^{32,34}$, Ca^{40}, and Ti^{48}. Deuteron energy spectra were taken over a range generally corresponding to 5 or 10 MeV of excitation of the final nucleus. Angular distributions were taken over a range from 6° to 60° for most of the well-defined deuteron groups. Reduced widths were extracted using the Butler theory. The $d_{3/2}-s_{1/2}$ and $d_{3/2}-d_{5/2}$ hole-energy differences in Ca^{39} were found to be 2.6 ± 0.15 and 6.3 ± 0.15 MeV, respectively. The $p_{1/2}-p_{3/2}$ difference in O^{15} was measured to be 6.4 ± 0.15 MeV.

I. INTRODUCTION

THE usefulness of direct processes such as stripping and pickup for the study of nuclear states is well established. Certainly the most heavily exploited reaction of this type has been the (d,p) stripping reaction. A large amount of such stripping data has been compiled and analyzed by MacFarlane and French,[1] hereafter referred to as MF. This type of analysis has in many cases yielded valuable information concerning the structure of certain nuclear states. However, since the target nuclei in these experiments are in their ground states, the excited states of many nuclei cannot be studied by (d,p) experiments (Ca^{39}, for example). Furthermore, the Pauli principle prevents the "stripped-in" nucleon from going into a filled level, and thus information can be obtained only about states which lie above the Fermi sea. In pickup reactions, such as (p,d), (p,t), (d,He^3), etc., the reverse situation is encountered, i.e., one is studying the "hole-excitations" of the final nucleus. Since from the point of view of nuclear theory, the hole states are as important as the particle states, the pickup reaction forms a useful complement to the stripping reaction.

Of the various types of pickup processes, the (p,d) reaction is probably the easiest to interpret theoretically. Reactions such as (d,t) and (d,He^3) yield angular distributions very similar to those observed in (p,d) transitions between the same nuclei, but the analysis of the reduced widths is complicated by the presence of d-t and d-He^3 normalization factors, which must be evaluated empirically. Hamburger and others[2,3] have studied (d,t) reactions on elements from Li^6 to Mg^{26}. In a similar fashion, (d,t) reactions on nuclei around $A=50$ have been performed.[4,5] Cujec[6] has studied a number of nuclei in the $1d$-$2s$ shell by means of the (d,He^3) reaction.

Of the two methods of studying single-neutron hole levels, (p,d) and (p,np), the former clearly presents fewer experimental difficulties. Nonetheless, there are still many problems in a (p,d) study which do not present themselves in the inverse stripping experiment. For example, the large (negative) Q values of most (p,d) reactions (commonly <-10 MeV) require a moderately high-energy proton beam. Furthermore, one must detect low-energy deuterons in the presence of a highly penetrating and usually very intense proton background. It is understandable, therefore, that the amount of available (p,d) data is somewhat sparse in comparison to the (d,p) data. The studies which have been made involve mainly heavy and medium-heavy nuclei.[7,8] The principle (p,d) experiments performed on the lighter elements are those of Bennett,[9] Legg,[10] and Cavanagh *et al.*[11] The present investigation was undertaken to augment the existing information on nuclei in the interesting region where the $2s$-$1d$ shells are filling, and to extend the measurements to reactions which had not previously been studied because of insufficient proton energy.

II. EXPERIMENTAL TECHNIQUES

The proton beam for this experiment was supplied by the University of Minnesota Linear Proton Accelerator. Analysis of the reaction products was carried out by means of a 40-in. magnetic spectrometer. After momentum analysis, the scattered particles were detected by an array of eight $\frac{1}{16}$-in.-thick plastic scintillators, coupled to Dumont-6467 photomultiplier tubes. The thickness of the scintillators was chosen so that protons down to about 30 MeV in energy gave smaller pulses than deuterons of the same magnetic rigidity.

The signals from the eight detectors were fed into a routing chasis, mixed, amplified and then pulse height

* Supported in part by the U. S. Atomic Energy Commission.
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[1] M. H. MacFarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).
[2] A. I. Hamburger, Phys. Rev. **118**, 1271 (1960).
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[7] C. D. Goodman and J. B. Ball, Phys. Rev. **118**, 1062 (1959).
[8] C. D. Goodman, J. B. Ball, and C. B. Fullmer, in Proceedings of the International Symposium on Direct Interactions, Padua (to be published).
[9] E. F. Bennett, Phys. Rev. **122**, 595 (1961).
[10] J. C. Legg, Phys. Rev. **129**, 272 (1963).
[11] P. E. Cavanagh, C. F. Coleman, G. A. Gard, B. W. Ridley, and J. F. Turner, P. L. A. Progr. Report 1962, NIRL/R/24, p. 49.

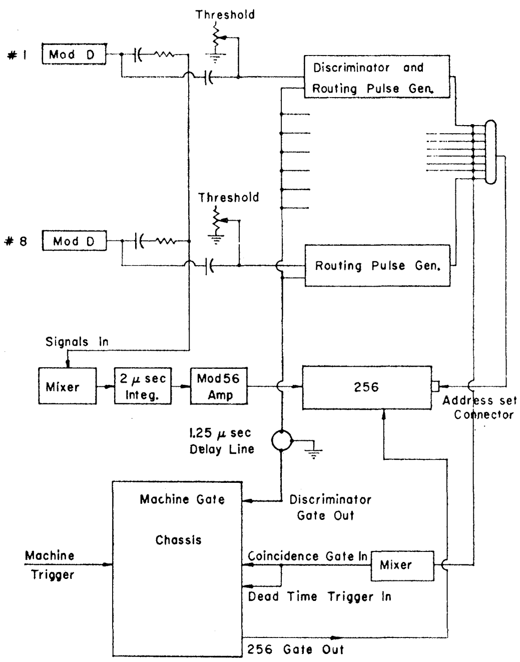

FIG. 1. Pulse-analyzing circuit.

analyzed by a Nuclear Data 256-channel analyzer. The operation of the ND 256 was modified to permit the output of each detector to be displayed in a separate octant of the analyzer. The routing chassis performs the function of directing a signal from a particular counter to the appropriate octant of the 256-channel analyzer. In this way, a 32-channel pulse-height spectrum was obtained for each detector, which greatly facilitated the separation of particles of the same magnetic rigidity. In order to reduce the background, the ND 256 was biased off during the "off time" of the Linac. Figure 1 shows the basic elements of the detection system.

Because of the rather large dead time of the pulse height-analyzer (about 85 μsec for the 8 input mode of operation), some method of correcting for counting losses had to be devised. A fast transistorized pulse-height analyzer was, therefore, placed in parallel with the ND 256 on the signal cable from one of the detectors. The observed ratio between the number of counts in the fast and slow analyzers then furnished an indication of counting losses for the detector in question. This same loss rate was then used to adjust the number of counts observed in each of the other seven octants of the ND 256.

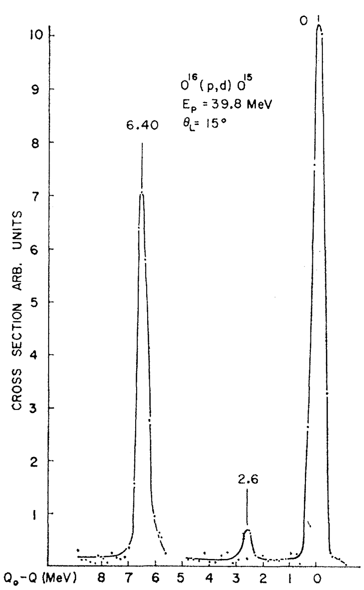

FIG. 2. Energy spectrum of deuterons from $O^{16}(p,d)$-O^{15}. The small group at 2.6 MeV is due to carbon contamination of the oxygen gas target.

III. PREPARATION OF TARGETS

The targets studied in this experiment were C^{12}, O^{16}, $Mg^{24,25,26}$, $S^{32,34}$, Ca^{40}, and Ti^{48}. The carbon target consisted of a 1.3-mg/cm² commercial polyethylene foil (CH_2). The $O^{16}(p,d)O^{15}$ reaction was studied by using a 1.77-mg/cm² Mylar foil ($C_{10}(H_2O)_4$) for the angular distributions of the ground and 6.4-MeV states. An oxygen gas target was used to investigate the region obscured by the ground-state reaction on the carbon contained in the Mylar.

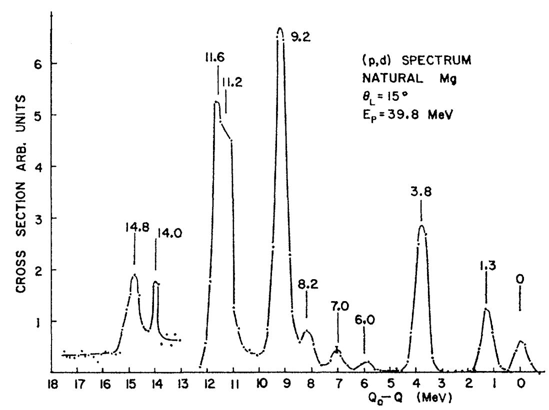

FIG. 3. Energy spectrum of deuterons from (p,d) reactions on natural magnesium.

The sulfur target was prepared from natural sulfur by condensation of the vapor on 10 μin. nickel foil after a technique described by Nielsen and Weinstein.[12] The three magnesium isotopes were observed using a 35.3-mg/cm² natural magnesium foil (78.8% Mg^{24}, 10.1% Mg^{25}, 11.1% Mg^{26}).

The preparations of the calcium target required special care because of the rapidity with which this material oxidizes. The ground state Q-values for Ca^{40} and O^{16} are quite close to one another (−13.58 and −13.43 MeV, respectively), thus making it imperative that a very clean calcium target be obtained. The

[12] E. Nielsen and A. Weinstein, Rev. Sci. Instr. 24, 1146 (1953).

TABLE I. Summary of results.

Target	J	T	Final	J	T	Q	E^*(MeV)	Δl	θ^2	Shell	S[a]	θ_0^2
C^{12}	0+	0	C^{11}	$\frac{3}{2}-$	$\frac{1}{2}$	−16.5	0	1	0.10	$p_{3/2}$	4	0.025[b]
O^{16}	0+	0	O^{15}	$\frac{1}{2}-$	$\frac{1}{2}$	−13.4	0	1	0.15	$p_{1/2}$	4	0.037
				$\frac{3}{2}-$	$\frac{1}{2}$	−19.8	6.4	1	0.22	$p_{3/2}$	8	0.028
Mg^{24}	0+	0	Mg^{23}	$\frac{3}{2}+$	$\frac{1}{2}$	−14.3	0	2	0.20	$d_{5/2}$	20	0.01[b]
Mg^{25}	$\frac{5}{2}+$	$\frac{1}{2}$	Mg^{24}	0+	0	−5.1	0	2	0.012	$d_{5/2}$	$\frac{1}{3}$	0.036
				2+	0	−6.4	1.3	2	0.034	$d_{5/2}$	⋯	⋯
S^{32}	0+	0	S^{31}	$\frac{1}{2}+$	$\frac{1}{2}$	−12.8	0	0	0.077	$2s_{1/2}$	4	≥0.019
				$(\frac{3}{2}+)$	$\frac{1}{2}$	−14.0	1.2	2	0.041	$d_{5/2}$	12	≥0.014
				$(\frac{5}{2}+)$	$\frac{1}{2}$	−15.2	2.4	2	0.13			
				$(\frac{1}{2}-)$	$\frac{1}{2}$	−16.2	3.4	1	0.025	$p_{1/2}$	⋯	⋯
S^{34}	0+	1	S^{33}	$\frac{3}{2}+$	$\frac{1}{2}$	−9.2	0	2	0.076	$d_{3/2}$	⋯	⋯
				$\frac{1}{2}+$	$\frac{1}{2}$	−10.0	0.8	0	0.11	$2s_{1/2}$	4	0.028
Ca^{40}	0+	0	Ca^{39}	$\frac{3}{2}+$	$\frac{1}{2}$	−13.6	0	2	0.085	$d_{3/2}$	8	0.011
				$(\frac{1}{2}+)$	$\frac{1}{2}$	−16.2	2.6	0	0.038	$2s_{1/2}$	4	0.011
				$(\frac{1}{2}+)$	$\frac{1}{2}$	−17.5	3.9	0	0.006			
				⋯	⋯	−19.9	6.3	2	0.018	$d_{5/2}$	12	0.0015
				⋯	⋯	−21.9	8.3	2+(?)	⋯	⋯	⋯	⋯
Ti^{48}	0+	2	Ti^{47}	$(\frac{7}{2}-)$	$\frac{3}{2}$	−9.4	0.16	3	0.127	$f_{7/2}$	8.5	0.015[b]

[a] The S assignments (and, hence, also the θ_0^2) are to be regarded as tentative only. The values given here are normally those expected on the basis of a pure shell-model configuration.
[b] These values of θ_0^2 have been estimated using the data presented in Ref. 1 in order to arrive at an approximate value for the spectroscopic factor.

technique finally adopted consisted in scraping the oxide off commercially available calcium foils while under ether, and immediately transferring the foils to the scattering chamber. The film of ether which adheres to the foil greatly retarded the oxidation which otherwise occurred during the time it took to evacuate the chamber. A very sensitive test for oxygen contamination could then be applied, which consisted of observing protons elastically scattered from the target at a scattering angle of 60°. Past experience with contaminated targets showed that center-of-mass effects at this angle were large enough to give two clearly separated proton groups, corresponding to scattering off O^{16} and Ca^{40}, respectively. The contamination of the targets used in this experiment is estimated to be less than 0.5%. The target area density was 15.7 mg/cm² with a nonuniformity (introduced mainly by the scraping process) of about 5%.

The reaction on Ti^{48} was studied using a 20-mg/cm² metallic foil target enriched to 98.86% in Ti^{48}.

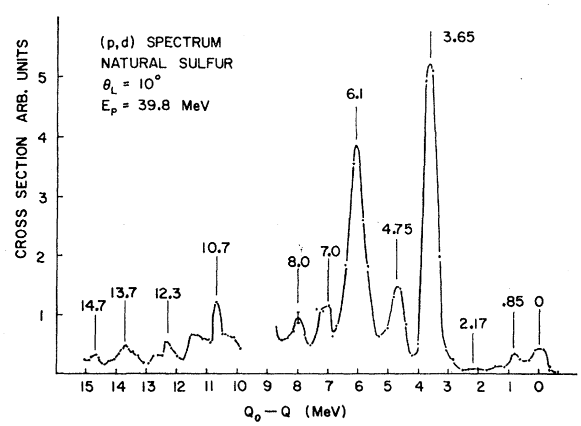

FIG. 4. Energy spectrum of deuterons from (p,d) reactions on natural sulfur.

IV. DISCUSSION OF RESULTS

A. Energy Spectra

Figures 2–5 show the deuteron-energy spectra obtained with targets of oxygen, natural magnesium, natural sulfur, and calcium. In the cases of carbon and

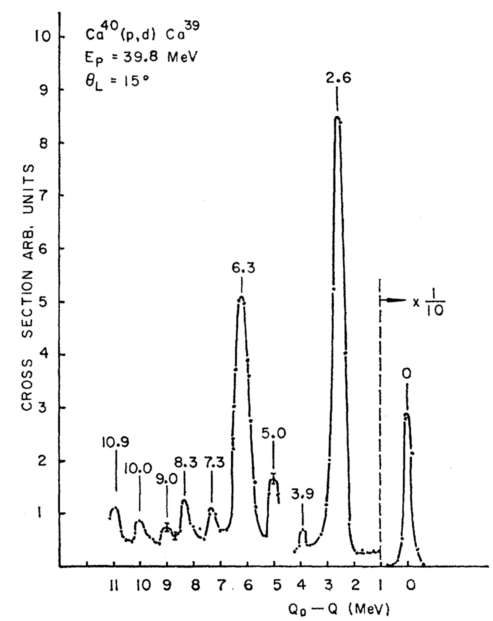

FIG. 5. Energy spectrum of deuterons from $Ca^{40}(p,d)Ca^{39}$.

titanium, we were interested only in a single transition and, hence, have not taken energy spectra for these two nuclei. The cross sections have been plotted against Q-Q_0, where in each figure, Q_0 refers to one of the following ground-state reactions: $O^{16}(p,d)O^{15}$ (−13.4 MeV), $Mg^{25}(p,d)Mg^{24}$ (−5.1 MeV), $S^{34}(p,d)S^{33}$ (−9.2

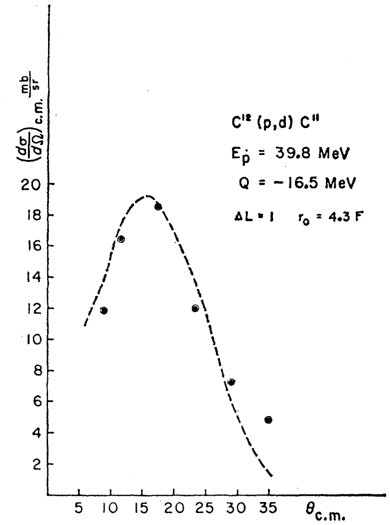

FIG. 6. Angular distribution of deuterons from $C^{12}(p,d)$-C^{11}, ground state.

MeV), $Ca^{40}(p,d)Ca^{39}$ (−13.6 MeV). The Q-value determinations of the excited states are probably accurate to within ±150 keV, and are determined relative to the ground-state values listed in published tables. The energy uncertainty is due mainly to the energy spread in the linac beam. For the targets which contained

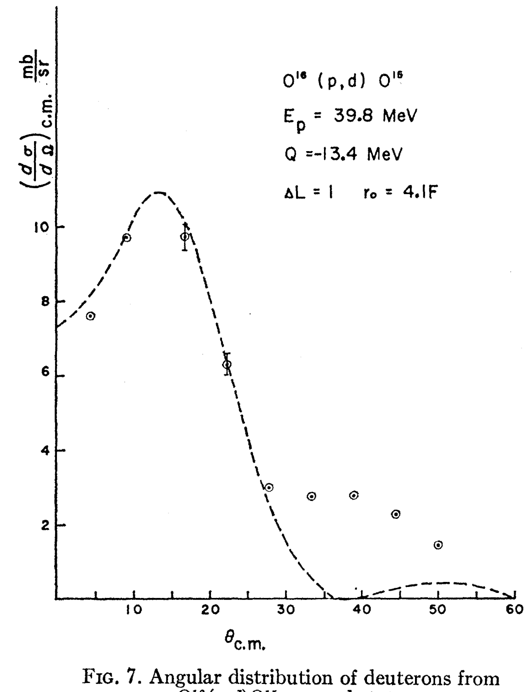

FIG. 7. Angular distribution of deuterons from $O^{16}(p,d)O^{15}$, ground state.

more than a single isotope in significant amounts (magnesium and sulfur), it was necessary to ascertain which deuteron groups corresponded to a particular isotope. For all of the strong groups studied, this could be done unambiguously. The identification is indicated by the entries in Table I under the heading "E^*."

B. Angular Distributions

Angular distributions from 6° lab to approximately 55° lab were taken at 5° intervals for most of the well-defined states observed in the spectra of Figs. 2–5. In addition, angular distributions for the reactions $C^{12}(p,d)C^{11}$(g.s.) and $Ti^{48}(p,d)Ti^{47}$ (0.16 MeV) were taken to ascertain the shapes of $l=1$ and $l=3$ transitions, respectively. It was found that the angular range studied here was sufficient to determine the l value of the transition, since normally the first and second

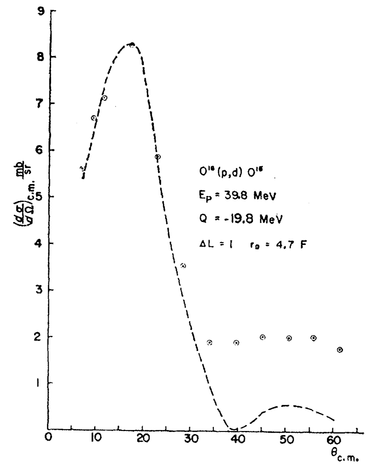

FIG. 8. Angular distribution of deuterons from $O^{16}(p,d)$-O^{15*}, 6.4-MeV state.

maxima could be observed. For the one case in which large angle data was taken [$Ca^{40}(p,d)Ca^{39}$g.s.], no backward peaking was observed up to 145° lab. In Legg's study[10] of (p,d) reactions at $E_p=20$ MeV on a number of nuclei around O^{16}, no evidence of back-angle peaking was found for any of the reactions considered.

The absolute cross sections are probably good to within ±20%. Relative cross sections should be much better than this, however. A relative error of ±10% is applicable to most of the reactions studied, and is determined mainly by the counting statistics. The

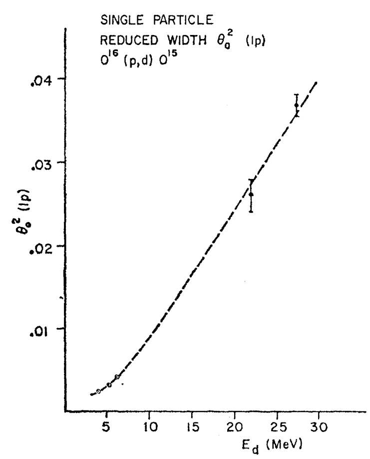

FIG. 9. Dependence of the single-particle reduced width $\theta_0^2(1p)$ on deuteron bombarding energy as measured from $O^{16}(p,d)O^{15}$. Open circles are data of Ref. 10.

sample errors shown reflect these statistical uncertainties only, and do not include uncertainties introduced by the normalization procedure.

Fits were made to the experimental data using the Butler theory, and are shown as dashed lines on Figs. 6–23. The calculations were carried out using a Fortran code for the CDC-1604 computer. The procedure followed was to run the code for each reaction with r_0 values spaced at intervals of 0.2 F and then to select the set of data which gave the best fit by eye

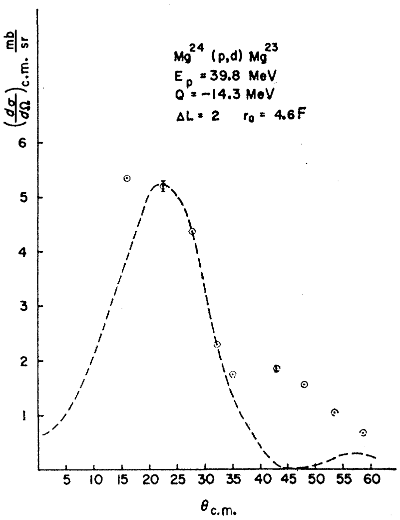

FIG. 10. Angular distribution of deuterons from Mg^{24}-(*p*,*d*)Mg^{23}, ground state.

to the experimental points. Reduced widths were then extracted using the method given by MF and are listed, together with other pertinent data, in Table I. In view of the uncertainties in the values for the absolute cross sections, it is estimated that the values for θ^2 are accurate to $\pm 25\%$.

In order to compare our results with those of MF, we have estimated spectroscopic factors in cases where the shell model is expected to give a good description of the initial and final nuclei. Single particle reduced widths θ_0^2 were then calculated from $\theta^2 = S\theta_0^2$ and com-

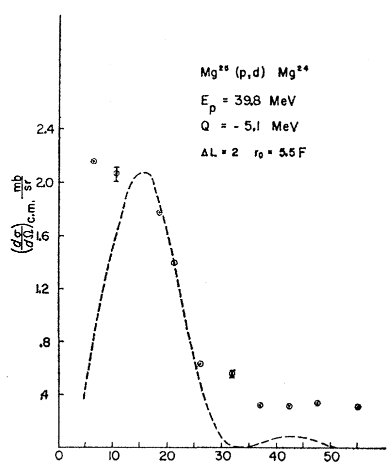

FIG. 11. Angular distribution of deuterons from Mg^{25}-(*p*,*d*)Mg^{24}, ground state.

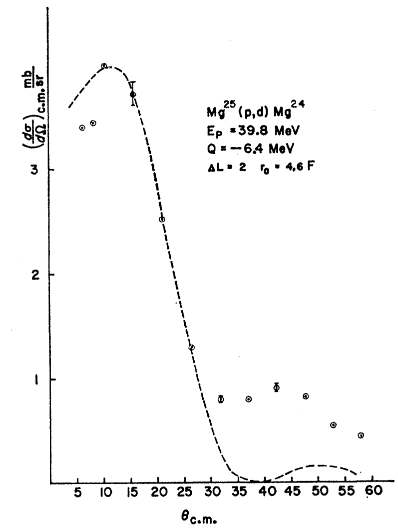

FIG. 12. Angular distribution of deuterons from Mg^{25}-(*p*,*d*)Mg^{24*}, 1.3-MeV state.

pared with extrapolations of curves found in Ref. 1 to the appropriate neutron binding energy. In most cases, the agreement was rather good, indicating that this type of analysis can be extended to bombarding energies outside the "moderate-energy" range (10 to 20 MeV) considered by MF. The tendency for θ_0^2 to decrease with increasing binding energy of the transferred nucleon was also well borne out.

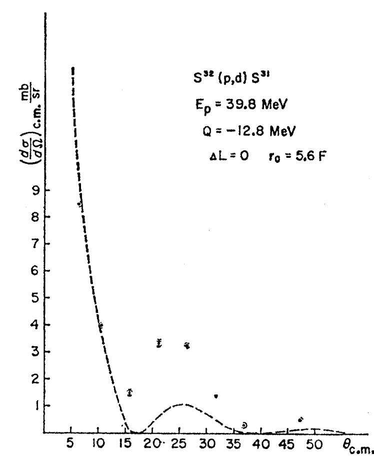

FIG. 13. Angular distribution of deuterons from S^{32}-(*p*,*d*)S^{31}, ground state.

C. C^{12}(*p*,*d*)C^{11} g.s.

A carbon target in the form of a thin 1.3-mg/cm^2 polyethylene foil was used to obtain the angular distribution for the ground-state transition shown in Fig. 6. The experimental points are quite well fitted by the Butler curve for $l=1$ with a cutoff radius of 4.3 F, indicating that the transition is due to the pickup of a *p* neutron. The reduced width was measured to be $\theta^2 = 0.10$.

For reactions where the spectroscopic factor S could be estimated, we have found reasonable agreement with the data of MF (see below). We thus feel justified in performing the inverse calculation for this reaction,

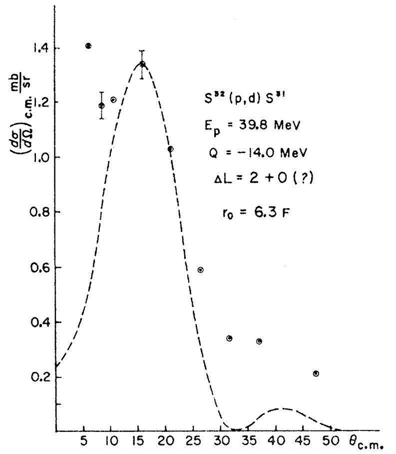

FIG. 14. Angular distribution of deuterons from S^{32}-$(p,d)S^{31*}$, 1.2-MeV state.

that is, to estimate $\theta_0^2(1p)$ from MF, and then to use this value to calculate S. Using the value $\theta_0^2(1p)=0.025$, we obtain $S=4$. This is to be compared with the j-j coupling prediction of $S=8$ which would apply if both initial and final states were pure shell-model configurations. Roughly speaking then, the C^{11} ground state has a configuration corresponding to a $p_{3/2}$ hole in the C^{12} ground-state wave function only 50% of the time. The remaining 50% of the $p_{3/2}$ hole state must appear at higher excitation energies. This conclusion is supported

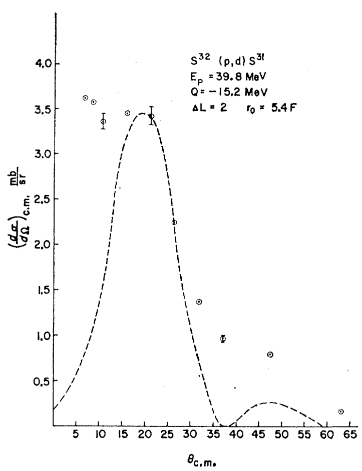

FIG. 15. Angular distribution of deuterons from S^{32}-$(p,d)S^{31*}$, 2.4-MeV state.

by recent data of Radvanyi *et al.*,[13] who observe a strong (p,d) transition to a $\frac{3}{2}-$ state at 4.8 MeV in C^{11}.

D. $O^{16}(p,d)O^{15}$ g.s. and 6.4-MeV State

The two strong deuteron groups observed in the oxygen spectrum displayed good $l=1$ angular distributions (see Figs. 7 and 8) and undoubtedly involve neutron pickup from the $p_{1/2}$ and $p_{3/2}$ shells, respectively. The reduced widths for these two transitions were measured to be θ^2 (g.s.) = 0.15 and θ^2 (6.4 MeV) = 0.22.

[13] P. Radvanyi, J. Genin, and C. Detraz, Phys. Rev. **125**, 295 (1962).

If the pure j-j coupling shell-model description of the initial and final nuclear states in these transitions were valid, the spectroscopic factors should be $S=4$ (g.s.) and $S=8$ (6.4 MeV). These values together with the reduced widths given above yield two determinations of θ_0^2 $(1p)$ (0.15 and 0.22), corresponding to the two different binding energies for the transferred neutron. One would then conclude that the single-particle reduced width θ_0^2 decreases by approximately 25%

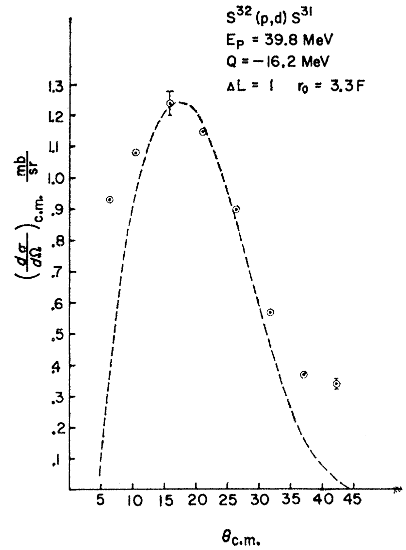

FIG. 16. Angular distribution of deuterons from S^{32}-$(p,d)S^{31*}$, 3.4-MeV state.

when the Q value of the reaction becomes 6 MeV more negative.

Another point of view that could be taken is to assume θ_0^2 $(1p)$ does not change much with Q, and to use the observed ratio θ^2 (6.4 MeV)/θ^2 (g.s.) = 1.5 to infer that there was some departure from the shell-model description of O^{16} and O^{15}. Which of the above procedures is the more nearly correct is open to question at the present time.

Legg[10] in a recent paper has adopted the point of

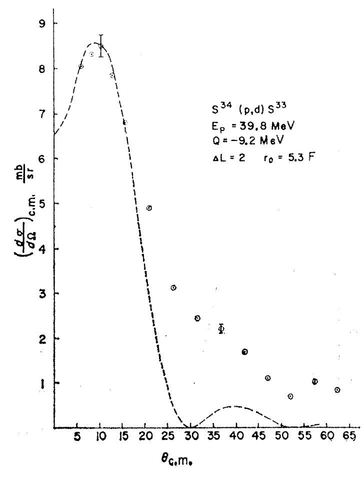

FIG. 17. Angular distribution of deuterons from S^{34}-$(p,d)S^{33}$, ground state.

view that both the ground states of O^{16} and O^{15} have good independent particle shell-model wave functions, and that the spectroscopic factor for the ground-state transition has the full value $S=4$. In this way, he obtains values for θ_0^2 ($1p$) that vary from 0.0024 at $E_p=18$ MeV to 0.0042 at $E_p=20$ MeV. On the basis of other experiments done in this mass region, one expects single-particle reduced widths to lie in the range 0.04–0.06 for moderate deuteron bombarding

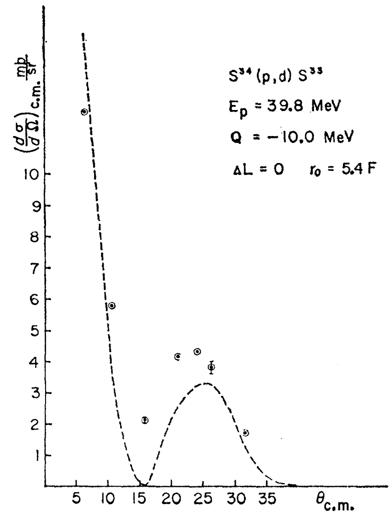

FIG. 18. Angular distribution of deuterons from S^{34}-(p,d)S^{33*}, 0.8-MeV state.

energies (7–15 MeV). Although the experiment of Ref. 10 was carried out at a proton energy corresponding to an E_d slightly outside this range ($E_d=6.1$ MeV), the author felt that an experiment done at $E_p=25$–30 MeV would not yield a value for θ_0^2 much higher than 0.01. This would indicate a very strong dependence of θ_0^2 on the binding energy of the transferred nucleon, since other experiments done in the $1p$ shell involved binding energies of less than 10 MeV (absolute value). The neutron binding energy in $O^{16}(p,d)O^{15}$ g.s. is 15.65 MeV.

The results of the present experiment indicate that if θ_0^2 ($1p$) remains approximately constant at 0.01 over

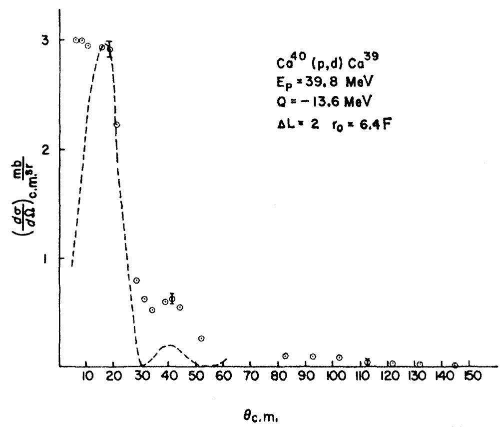

FIG. 19. Angular distribution of deuterons from $Ca^{40}(p,d)Ca^{39}$, ground state.

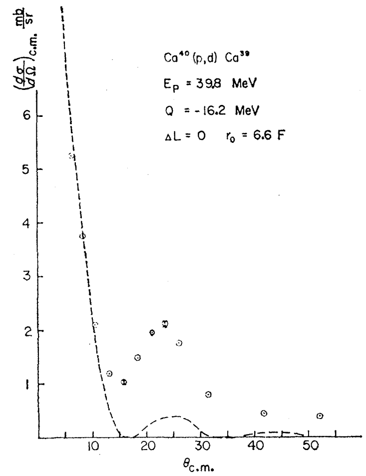

FIG. 20. Angular distribution of deuterons from Ca^{40}-(p,d)Ca^{39*}, 2.6-MeV state.

the range 7 MeV $\leq E_d \leq 15$ MeV, as Legg suggests, than it must again increase sharply in order to obtain a value of 0.037 at $E_d=27.4$ MeV (the deuteron energy in the present experiment). This sort of behavior appears unlikely, since an experiment carried out at a proton energy corresponding to $E_d=21.9$ MeV (using the 40-MeV linac beam degraded by absorbers) yielded a value $\theta_0^2=0.026$. In fact, a linear dependence of θ_0^2 ($1p$) on bombarding energy cannot be ruled out (see Fig. 9).

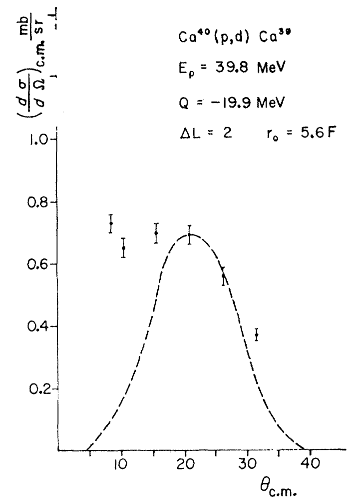

FIG. 21. Angular distribution of deuterons from Ca^{40}-(p,d)Ca^{39*}, 6.3-MeV state.

E. $Mg^{24}(p,d)Mg^{23}$

Figure 10 shows the angular distribution for the ground-state transition and furnishes the first of several examples of $l=2$ angular distributions which do not turn down at small angles as the Butler theory would predict (see also Figs. 11, 14, 15, and 19). A similar effect has been observed by other experimentors, and had previously been regarded as an indication of an $l=0$ admixture in the angular distribution.[14] However,

[14] H. E. Gove, in *Proceedings of the Rutherford Jubilee International Conference*, 1962 (Heywood and Company Ltd., Manchester, 1962), p. 447.

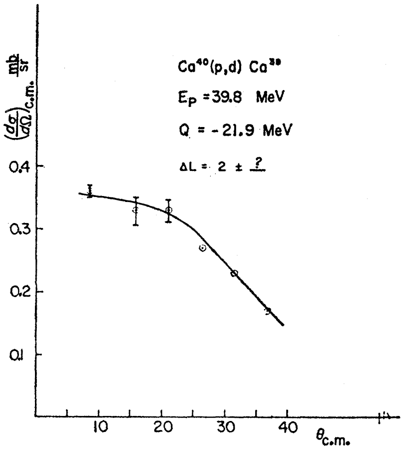

FIG. 22. Angular distribution of deuterons from Ca^{40}-$(p,d)Ca^{39*}$, 8.3-MeV state.

distorted-wave Born approximation calculations employing only $l=2$ have been able to fit the small-angle data in many of these cases by using a Gaussian shape for the imaginary part of the deuteron potential. Furthermore, the effect is observed in cases where the initial and final spins and parities are such as to forbid any $l=0$ contribution. Consequently, no attempt has been made in these "anomalous" cases to fit the small-angle data by superimposing an $l=0$ Butler curve.

Mg^{23} is expected to be an appreciably distorted nucleus, and consequently the pure independent shell-model prediction of the spectroscopic factor is not meaningful. Furthermore, the reduced width for this reaction appears to be unusually high, since if one assumes a value of θ_0^2 $(1d)\cong 0.01$ (as estimated from MF), a value of $S=20$ is obtained. This is almost a factor of two larger than the maximum value predicted for pickup from the $d_{5/2}$ shell.

F. $Mg^{25}(p,d)Mg^{24}$, g.s., 1.34 MeV

The angular distributions for these two reactions are shown in Figs. 11 and 12. These two reactions and the (d,t) analogs have been studied by Bennett[9] and Hamburger and Blair.[3] Bennett finds reduced widths of 0.0079 and 0.022, respectively, which are somewhat smaller than the values determined from the present experiment (0.012 and 0.034). However, the ratio-to-ground state is the same for both experiments (2.8). The smaller values in the Bennett experiment ($E_p=17$ MeV) may reflect the tendency noted in MF for the single-particle reduced widths to increase with bombarding energy.

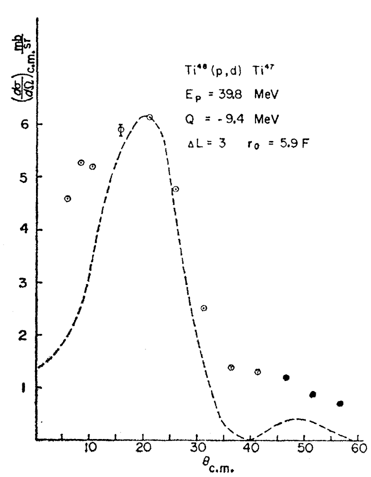

FIG. 23. Angular distribution of deuterons from Ti^{48}-$(p,d)Ti^{47*}$, 0.16-MeV state.

The reduced widths measured in the (d,t) transitions between the same states were 0.0085 and 0.017, giving a ratio θ^2 (1.37 MeV)/θ^2 (g.s.)$=2.0$. However, the analysis of the (d,t) data is complicated by the presence of an additional parameter, B^2, the triton normalization factor. We believe that the (p,d) results are the more reliable of the two sets of measurements.

It is interesting to note that the "anomalous" small-angle behavior discussed earlier is observed for the ground state transition (Fig. 11), but not for the transition to the 1.3-MeV state (Fig. 12). This was also the case with the (d,t) experiment.

The ground state of Mg^{25} is well described by the rotational model, and a calculation done in Ref. 1 gives the value $S=\frac{1}{3}$ for ground-state transition. This then gives a value for θ_0^2 $(1d)$ of 0.036, which is about 20% higher than the value given in MF for the lower bombarding energies considered there. This may display a tendency for θ_0^2 to increase with bombarding energy.

G. $S^{32}(p,d)S^{31}$

The angular distribution for the ground-state transition is shown in Fig. 13 and displays a well-defined $l=0$ shape, indicating that a neutron is being picked up from the $2s_{1/2}$ shell. The calculation of the reduced width for this and other $l=0$ transitions is complicated by the fact that it is not obvious where one should normalize the Butler curve to the experimental results. For the sake of consistency, we have followed the convention used in other pick-up and stripping investigations,[3,9,10] and have used the small-angle data ($\theta<15°$) to extract θ^2. When this is done, a value of $\theta^2=0.077$ is obtained. It is not possible in this case to calculate the single-particle reduced width θ_0^2 $(2s_{1/2})$ since there is evidence from other experiments that the ground state of S^{32} contains an appreciable amount of core excitation.[6,15] Consequently, the spectroscopic factor for the ground-state transition is likely to be somewhat smaller than the j-j coupling value of 4, so that the only statement that can be made is that θ_0^2 $(2s_{1/2})$ >0.019. An extrapolation of the MF data to the binding energy of the transferred neutron appropriate to this experiment (15.08 MeV) would lead one to expect a value θ_0^2 $(2s_{1/2})=0.01$–0.02.

From the appearance of the angular distribution for the reactions leaving S^{31} in the 1.2- and 2.4-MeV states

[15] M. H. MacFarlane and J. B. French, Rev. Mod. Phys. **32**, 573 (1960).

(Figs. 14 and 15), it is clear that the main contribution comes from an $l=2$ transition. Both distributions show a rise at forward angles in contrast to the Butler prediction. This is probably an effect similar to that observed in $Mg^{25}(p,d)Mg^{24}$, g.s. since the initial and final parities and spins of the states involved are such as to forbid any $l=0$ contribution.

From the $l=2$ nature of this angular distribution for these two reactions, it is inferred that a $d_{5/2}$ neutron is being picked up. The absence of any other strong $l=2$ groups below 3.5-MeV excitation in S^{31}, and the fact that in the region of excitation above this (up to 11 MeV) only weak states were seen leads us to believe that the bulk of the $d_{5/2}$ hole state in S^{31} has been located. Since the reduced width for the transition to the 2.3-MeV state (measured 2.4 MeV) is much larger than that for the reaction leading to the 1.1-MeV states (0.13 versus 0.041), it is clear that the "baricenter" of the $d_{5/2}$ hole state in S^{31} must lie quite near to 2.3 MeV. The j-j coupling prediction for the spectroscopic factor is $S=12$ for pickup from the closed $d_{5/2}$ shell. Allowing for the possibility that other small "pieces" of the $d_{5/2}$ hole state have been missed, we can give only a lower limit for θ_0^2 ($d_{5/2}$) of 0.014. This value is not inconsistent with an extrapolation of the MF data to a neutron binding energy of 17.4 MeV. The proton energy in this experiment corresponds to an E_d of about 24 MeV.

The state at 3.4 MeV required an unusually small cutoff radius (3.3 F) to fit the experimental points with an $l=1$ curve (see Fig. 16). Attempts were made to achieve a fit using $l=2$, but when one matched the position of the experimental maximum, this theoretical peak was found to be much too narrow. It is therefore believed that the l assignment is correct, indicating that the neutron is coming from the $p_{1/2}$ shell. Because of the small cutoff radius, however, probably one cannot attach much significance to the measured reduced width, $\theta^2=0.025$.

H. $S^{34}(p,d)S^{33}$

Because of the low relative abundance of S^{34} in the natural sulphur target (4.2%), deuterons from $S^{32}(p,d)S^{31}$ obscure most of the spectrum resulting from $S^{34}(p,d)S^{33}$. However, the ground-state transition and transitions to two excited states ($E^*=0.8$ and 2.2 MeV) can be observed because the Q values are lower (in absolute value) than that for $S^{32}(p,d)S^{31}$ g.s. Figure 17 shows on $l=2$ fit to the ground-state transition, and the degree of agreement indicates that the l-value assignment is correct. Thus, what is being observed is the pickup of a neutron from the half-filled $d_{3/2}$ shell. The relatively small reduced width for this transition, $\theta^2=0.076$, indicates that there are probably sizeable admixtures of other configurations present in the ground-state wave function.

The transition to the state at 0.8 MeV in S^{33} is fitted quite nicely by an $l=0$ Butler curve (Fig. 18) indicating that we are observing essentially the same process as in $S^{32}(p,d)S^{31}$ g.s. The fact that the reduced width for the latter reaction is smaller (0.077) than for the one under discussion here (0.11) is probably due to the tendency noted earlier for θ_0^2 to decrease with decreasing Q value.

I. $Ca^{40}(p,d)Ca^{39}$

In Ca^{40} we again have a doubly magic nucleus, and we assume, as for O^{16}, that the ground-state configuration is pure double-closed shell. The transition leading to the ground state of Ca^{39} shows the same "anomalous" sort of $l=2$ angular distribution as was observed in magnesium (See Fig. 19), and corresponds to the pickup of a neutron from the closed $d_{3/2}$ shell. However, the ground state of Ca^{39} is known to be $\frac{3}{2}+$, so that the only l value allowed for this reaction is $l=2$. The Butler fit was made to the large-angle side of the first "maximum," as was done for the other "anomalous" $l=2$ distributions. It is significant that no back-angle peaking in the experimental data was observed out to $\theta_{c.m.}=145°$, indicating that heavy-particle pickup does not play an important role here.

The reduced width for the ground-state transition was measured to be $\theta^2=0.085$. No other significant $l=2$ transitions were observed below 11 MeV of excitation of Ca^{39} (with the exception of the strong $l=2$ state at 6.3 MeV, which represents the $d_{5/2}$-hole state). Consequently, the spectroscopic factor for the ground-state transition should be very close to the full value $S=8$. This gives a value for the $2d$ single-particle reduced width of θ_0^2 $(2d_{3/2})=0.011$ at a binding energy for the transferred neutron of $B=15.8$ MeV. This may be slightly lower than an extrapolation of Fig. 57 of MF would give, but the degree of agreement is very good considering that the bombarding energy is well outside the moderate-energy range studied there.

The reaction leading to the 2.6-MeV state in Ca^{39} shows a well defined $l=0$ angular distribution (Fig. 20) and undoubtedly corresponds to pickup from the $2s_{1/2}$ shell. Since the only other $l=0$ transition observed up to 11 MeV of excitation in Ca^{39} was the very weak state at 3.9 MeV ($\theta^2=0.006$ as compared to $\theta^2=0.038$ for the 2.6-MeV state), we feel reasonably confident in giving the position of the $2s_{1/2}$ hole state in Ca^{39} as 2.6 ± 0.15 MeV. Furthermore, the spectroscopic factor for the transition should have the j-j coupling value $S=4$ when both states are considered together. Assuming θ_0^2 $(2s)$ does not change much for a change in Q value of 1.3 MeV, and adding the reduced widths for the two transitions, one obtains θ_0^2 $(2s)=0.011$. This result also agrees with an extrapolation of the MF data to the appropriate value of B (18.4 MeV).

An angular distribution for the state at 5.0-MeV excitation could not be obtained because of interference from the tail of the elastic proton peak (see Ca spectrum, Fig. 5).

Initial measurements made on the state at 6.3 MeV were complicated by the presence of a sizeable oxygen contamination. However, by refining the target preparation procedure, the oxygen contribution was reduced to insignificance. There still remained, however, the problem of resolving deuterons from the (more numerous) inelastic protons which have the same Br. In spite of this difficulty, the main feature of the observed angular distribution for this transition (Fig. 21) is its $l=2$ character (compare Fig. 19). Here, again, as for the ground-state transition, there is a pronounced "filling-in" at small angles due to distortion effects in the projectile wave functions. The intensity of this state is much higher than that of any other group at this excitation and is believed to contain most of the $d_{5/2}$ hole state. This means that the spin-orbit splitting $d_{3/2}$–$d_{5/2}$ has the value 6.3±0.15 MeV in Ca^{39}. From the measured reduced width, $\theta^2=0.018$, and the predicted value for the spectroscopic factor, $S=12$, one obtains θ_0^2 $(1d)=0.0015$. This value is almost a factor of ten smaller than the value one obtains from the ground-state transition, a fact which probably reflects the strong dependence of θ_0^2 on the Q value noted earlier.

Because of low intensities of the states and experimental difficulties arising from separation of deuterons from inelastic protons, angular distributions on transitions to higher excited states of Ca^{39} were not regarded as being meaningful. Figure 22 shows the typical situation that was encountered; namely, a monotonic decrease in angle with no characteristic oscillatory behavior. If states in this region subsequently turn out to be of interest, it would be possible to study them fairly easily by employing the 68-MeV proton beam, since then approximately 20 MeV of excitation of Ca^{39} could be observed without encountering proton background.

J. $Ti^{48}(p,d)Ti^{47}$

The reaction observed here is very likely a transition to the 0.16-MeV level in Ti^{47} rather than to the ground state, since the ground-state transition was not observed in a recent (d,t) study.[5] The ground-state spin of Ti^{47} is $(\frac{5}{2})$ and one would not expect to form this state by pickup of an $f_{7/2}$ neutron from Ti^{48}. The shape of the angular distribution indicates that $l=3$ for the transition, but the fit is not outstanding. The main trouble seems to be that width of experimental first maximum is somewhat greater than that for the Butler curve, if one normalizes at $\theta_{c.m.}=22°$. A similar difficulty with $l=3$ shapes was observed in a (d,p) study by Hoogenboom *et al.*[16] For purposes of calculating the reduced width, we have followed the usual convention of fitting the data at the experimentally determined peak of the first maximum ($\theta_{c.m.}=22°$). Estimating a value θ_0^2 $(1f)\cong 0.015$ from MF, we obtain a value for the spectroscopic factor of $S=8.5$, as opposed to the independent shell-model prediction $S=12$.

V. CONCLUSIONS

From the nature of the data which has been presented, it appears that useful information can be obtained about the location of the single-particle shell-model states in nuclei even with moderate energy resolution. This is made possible by the high degree of selectivity of the pickup reaction, resulting in excitation of only few of the many levels of the final nucleus. The method of extracting such information from the data of this experiment involved the Butler analysis, whose usefulness depends upon several critical assumptions. The first of these is that by proper choice of a cutoff radius r_0, the shape of the experimental angular distribution can be fitted, and the l value for the transition identified. Furthermore, this l-value determination must be unambiguous.

The bulk of the preceding results supports such an assumption. That is, for all angular distributions which showed characteristic oscillatory behavior, an unambiguous l-value determination could be made with a reasonable r_0 value.

The second assumption one must make in the Butler analysis is that the bulk of the reaction mechanism effects are extracted by the theory. This means, in particular, that the parameter θ_0^2 is only slowly varying with Q and the bombarding energy. In this way, a determination of θ_0^2 for some region of the Periodic Table can lead to information about the spectroscopic factors for nuclear reactions in this region. Unfortunately, this assumption is not consistently borne out by the data. There is evidence of a strong dependence of θ_0^2 on the Q value of the reaction, for example. However, this dependence seems to be smooth and, to a great extent, predictable. Furthermore, a comparison of the present results to those compiled in MF indicates the effect of bombarding energy is not a serious one. The one exception to this (the case of the reaction on oxygen) involves a comparison of data obtained by different experimenters at different bombarding energies and is not conclusive.

[16] A. M. Hoogenboom, E. Kashy, and W. W. Buechner, Phys. Rev. **128**, 305 (1962).